# Locally compact groups beyond Lie theory

Spa, Belgium

Sunday, March 31, 2013
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Saturday, April 6, 2013

**abstracts of lectures and posters**

## Monday, 1st April 2013

**Uri Bader**
(Technion)

*09:00 – 09:50*

*Semi-simple theory beyond Lie theory*
(based on a joint project with Alex Furman)

In classical semi-simple theory one attaches certain combinatorial data to a Lie group: a root system, its Weyl group and the associated Coxeter complex. The latter forms the shape of the apartments in the Tits building of the group.

Using ergodic theoretical tools we can extend some of the above notions and define them for any locally compact group. Our definitions seem to give trivial information in "most" cases (rank 1 behaviour), but in the cases they don't (higher rank) the information appears to be useful.

In my talks I will explain the precise kind of data we associate to a given group and how we use it to control its representations.

**Misha Ershov**
(UVirginia)

*10:00 – 10:50*

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## *Golod–Shafarevich groups and their relatives*

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A group is called Golod–Shafarevich if it admits a presentation with a small set of relators (where relators are counted with different weights depending on how deeply they lie in the Zassenhaus filtration). These groups were introduced almost 50 years ago as a tool for solving two outstanding problems —class field tower problem and the general Burnside problem— and since then have beed used to settle many other interesting questions.

I will give a brief survey of the main structural properties and applications of Golod–Shafarevich groups. I will also compare Golod–Shafarevich groups with other classes of groups which behave similarly in many ways, including groups of positive $p$-gradient and groups of positive $p$-deficiency.

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**Tim Steger** *11:30 – 12:20*

(Sassari)

*The unitary dual of the automorphism group of a homogeneous tree*

Following the work of A. Yu. Olshanskii and the book of A. Figà-Talamanca and C. Nebbia, I will discuss the representations making up the above unitary dual; also the form of the Plancherel measure. The group in question is a relatively easy example. Its unitary dual is similar to that of $\mathsf{SL}(2, \mathbb{R})$ and even closer to that of $\mathsf{SL}(2, \mathbb{Q}_p)$.

**Ben Green** *15:00 – 15:50*
(Cambridge)

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*The structure of approximate groups and Hilbert's 5th problem, part I*

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**Emmanuel Breuillard** *16:30 – 17:20*
(Orsay)

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*The structure of approximate groups and Hilbert's 5th problem, part II*

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We will describing the structure theorem for approximate groups. After presenting the context of the statement, we will present some details of the proof, emphasizing the relation with Hilbert's fifth problem for locally compact groups and the role of the Gleason–Yamabe lemmas. If time permits we will describe further geometric applications of the main theorem.

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## Tuesday, 2nd April 2013

**Uri Bader** *09:00 – 09:50*
(Technion)

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**Misha Ershov** *10:00 – 10:50*
(UVirginia)

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**Tim Steger** *11:30 – 12:20*
(Sassari)

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See p. 3–5 for title and abstracts.

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**Abderezak Ould Houcine** *15:00 – 15:50*

(Mons)

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*Finitely presented groups with infinitely many non-homeomorphic asymptotic cones*

(joint work with D. Osin)

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We construct a finitely presented group with infinitely many non-homeomorphic asymptotic cones. We also show that the existence of cut points in asymptotic cones of finitely presented groups does, in general, depend on the choice of scaling constants and ultrafilters.

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**Anand Pillay**
(Leeds)

*16:30 – 17:20*

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*Definable, or tame, topological dynamics*

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We develop a theory of actions of a group $G$ on compact spaces where $G$ is a definable group (in some structure). We want a theory which is analogous to the topological case and is a strict generalization of the discrete case. This is motivated by work of Newelski on trying to use notions of topological dynamics in model theory. We do a case analysis for $G = \mathsf{SL}(2, \mathbb{R})$, considered neither as a topological group, nor a discrete group, but as a semialgebraic group, that is a group definable in $(\mathbb{R}, +, \cdot)$.

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## Wednesday, 3rd April 2013

**Yiftach Barnea**
(London)

*09:00 – 09:50*

*Abstract commensurators of profinite groups*

(joint work with Mikhail Ershov and Thomas Weigel)

Every totally disconnect locally compact group $L$ contains a compact open subgroup $G$, i.e., an open profinite subgroup. It is natural to ask what is the connection between the structure of $G$ and the structure of $L$. In particular, what profinite group could be an open subgroup of a simple totally disconnect locally compact group? We will demonstrate how the abstract commensurator of $G$ could be a very useful tool in answering this question, both in a positive and a negative way.

**George Willis**
(Newcastle) *10:00 – 10:50*

*The scale on totally disconnected, locally compact groups*

The talk will contrast some similarities between the theory of totally disconnected, locally compact groups and Lie theory with some important differences.

The scale $s\colon G \to \mathbb{N}$ is a continuous function defined on any totally disconnected, locally compact group $G$. This function and auxiliary concepts have parallels with Lie theory: they are used to answer questions about totally disconnected groups that may be answered for connected groups by approximating by Lie groups; the scale may be calculated in terms of eigenvalues when $G$ happens to be a Lie group over a local field; and theorems about the scale are inspired by analogies with Lie theory.

A significant difference with Lie theory is that the contraction group of an automorphism of a totally disconnected, locally compact group is generally not closed. For each automorphism of $G$, there is an associated compact subgroup, called the *nub* of $\alpha$, that has no counterpart in Lie theory and is trivial if and only if the contraction group of $\alpha$ is closed.

**Colin Reid**
(Newcastle)

*11:30 – 12:20*

*Locally normal subgroups of simple locally compact groups*

(joint work with Pierre-Emmanuel Caprace and George Willis)

In this talk I will present some joint work with Pierre-Emmanuel Caprace and George Willis, in which we develop and apply new tools for studying totally disconnected, locally compact (t.d.l.c.) groups. We have found these methods to be particularly fruitful in the case in which the groups under consideration are compactly generated and topologically simple; in this context we obtain some general results, such as sufficient conditions (of a purely local nature) for the group to be abstractly simple, non-amenable, and/or non-uniscalar.

The central idea is that of a locally normal subgroup, that is, a compact subgroup with open normaliser. The set of locally normal subgroups of a t.d.l.c. group $G$ modulo commensurability naturally forms a modular lattice $LN(G)$, which is a local invariant of $G$ (that is, it is determined by the isomorphism type of any open subgroup). We also define two canonical Boolean lattices that occur as subposets of $LN(G)$, which arise from direct decompositions of locally normal subgroups. The action of $G$ on these lattices is then used to analyse the global structure of $G$.

## Thursday, 4th April 2013

**John Wilson**
(Oxford) *09:00 – 09:50*

## *Subgroups of finite index in branch groups*

Associated with each branch group $G$ is a certain descending chain $(R_n)$ of normal subgroups of finite index with trivial intersection. ($R_n$ is the product of the rigid stabilizers of all vertices in the $n$th layer of the tree on which $G$ acts). A basic and powerful property of branch groups $G$ is that each non-trivial normal subgroup contains the derived group of some $R_n$. We shall describe a related (but weaker) property of subgroups of $G$ with only finitely many conjugates. We shall also discuss applications, to abstract commensurability, and to the study of the structure lattices of branch groups and related groups.

**Helge Glöckner**
(Paderborn) *10:00 – 10:50*

## *Totally disconnected, locally compact contraction groups*

Let $G$ be a locally compact group admitting an automorphism $\alpha$ which is *contractive* in the sense that $\alpha^n(g) \to 1_G$ as $n \to \infty$, for each $g \in G$. In a classical work [4], Siebert showed that $G$ then is the internal direct product of its connected component $G_0$ and an $\alpha$-stable closed subgroup $D$ of $G$ which is totally disconnected. It therefore suffices to understand the two extreme cases that $G$ is either connected or totally disconnected. In the first case, $G$ is a simply connected, nilpotent real Lie group whose Lie algebra admits a positive graduation [4]. In the talk, I'll describe more recent results concerning the case that $G$ is totally disconnected. As shown in [3], the set $\mathrm{tor}(G)$ of torsion elements and the set $\mathrm{div}(G)$ of divisible elements are closed subgroup of $G$ in this case, and

$$G = \mathrm{tor}(G) \times \mathrm{div}(G).$$

Moreover, $\mathrm{div}(G) = G_{p_1} \times \cdots \times G_{p_n}$ is a direct product of $\alpha$-stable $p$-adic Lie groups $G_p$ for finitely many primes $p$ [3]. The structure of the Lie groups $G_p$ is known: They are $p$-adic rational points of suitable unipotent groups, and their Lie algebra admits a positive graduation [5]. Using invariant manifolds for time-discrete, analytic dynamical systems over local fields [2] as a tool, one can show that also in positive characteristic, Lie groups over such fields admitting a contractive analytic automorphism are nilpotent [1].

### References

[1] Glöckner, H. *Contractible Lie groups over local fields*, Math. Z. **260** (2008), 889–904.

[2] Glöckner, H., *Invariant manifolds for analytic dynamical systems over ultrametric fields*, to appear in Expo. Math., `http://dx.doi.org/10.1016/j.exmath.2013.01.009`.

[3] Glöckner, H. and G. A. Willis, *Classification of the simple factors appearing in composition series for totally disconnected contraction groups*, J. Reine Angew. Math. **634** (2010), 141–169.

[4] Siebert, E., *Contractive automorphisms on locally compact groups*, Math. Z. **191** (1986), 73–90.

[5] Wang, J. S. P., *The Mautner phenomenon for $p$-adic Lie groups*, Math. Z. **185** (1984), 403–412.

**Rögnvaldur Möller** *11:30 – 12:20*
(Reykjavik)

*Independence conditions for group actions on trees*
(joint work with Jan Vonk)

In 1970 Tits studied "property P" for group actions on trees. He showed that if $G$ is a group acting on a tree with property $P$ (and satisfying some non-triviality conditions) then the subgroup generated by the stabilizers of edges is simple. In this talk I want to discuss groups acting on trees that have the property that the stabilizer of every half-tree is non-trivial. These ideas can be used to extend (slightly) recent results of Caprace and De Medts. They can also be used in the study of automorphism groups of locally finite graphs with more than one end, in particular one can show that the full automorphism group of a locally finite primitive graph with more than one end has a subgroup of finite index that is topologically simple.

**Dennis Dreesen** *15:00 – 15:50*
(Southampton)

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*Locally compact convergence groups, $n$-transitive actions and embeddings into $L_p$-spaces*

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The common convention when dealing with hyperbolic groups is that such groups are finitely generated and equipped with the word length metric relative to a finite generating subset. Gromov's work already contained ideas which encompass locally compact hyperbolic groups. Following recent work of Caprace, de Cornulier, Monod and Tessera, we define a locally compact group $\Gamma$ to be **hyperbolic** if it is compactly generated and hyperbolic with respect to the word length metric relative to some compact generating subset. The most famous class of locally compact non-discrete non-elementary hyperbolic groups are the families $SO(n,1), SU(n,1)$ and $Sp(n,1)$.

It turns out that the locally compact case can admit very counter-intuitive behaviour. For example, although non-elementary discrete hyperbolic groups contain a free subgroup and are thus non-amenable, Caprace, de Cornulier, Monod and Tessera show that there *are* amenable non-elementary locally compact hyperbolic groups.

In this talk, we first elaborate on joint work with Mathieu Carette. We note that Bowditch's topological characterization of hyperbolicity persists in the locally compact setting. This leads to a breakthrough in the classification of sharply-$n$-transitive actions on compact spaces and leads to a characterization of non-elementary boundary transitive hyperbolic groups as non-compact convergence groups acting transitively on an infinite compact set.

Finally, we elaborate on embeddings of locally compact hyperbolic groups into $L_p$-spaces. This leads to the calculation of all equivariant $L_p$-compressions of $SO(n,1)$. Moreover, generalizing a result of G. Yu, independently also proven by M. Bourdon, we use hyperbolicity to give a new proof that $Sp(n,1)$ and $F_4^{-20}$ admit proper affine isometric actions on $L_p$-spaces for $p > 4n+2$ and $p > 22$ respectively.

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**Yves de Cornulier** *16:30 – 17:20*
(Orsay)

*On the quasi-isometry classification of focal hyperbolic groups*

An isometric action on a Gromov-hyperbolic space is *focal* if it fixes a unique boundary point and some element acts as a hyperbolic isometry. A compactly generated locally compact (CGLC) group is *focal hyperbolic* if it is Gromov-hyperbolic and the action on itself is focal. Such groups are non-unimodular and in particular are non-discrete and were characterized algebraically in a joint work with Caprace, Monod and Tessera. The quasi-isometry classification of focal hyperbolic CGLC groups boils down to two conjectures, describing on the one hand quasi-isometric classification within focal groups, and on the other hand the description of those focal groups that are quasi-isometric to a non-focal group; conjecturally the only such groups are quasi-isometric to a symmetric space or a tree. I will present partial results by various people towards these conjectures, mostly involving quasisymmetric homeomorphisms on the boundary spheres of homogeneous negatively curved Riemannian manifolds.

## Friday, 5th April 2013

**Dan Segal**
(Oxford)

*09:00 – 09:50*

*Normal subgroups of compact groups*

(joint work with Nikolay Nikolov)

We investigate non-closed normal subgroups in a compact group $G$, using both finite group theory and ultralimits. Applications include the theorem: if $G/N$ is finitely generated (as an abstract group) then $G/N$ is finite and $N$ is open.

**Shahar Mozes**
(Jerusalem)

*10:00 – 10:50*

*Simple locally compact groups without lattices*

(joint work with Uri Bader, Pierre-Emmanuel Caprace and Tsachik Gelander)

A classical theorem by Borel ensures that any simple Lie group contains both uniform and non-uniform lattices. The goal of this talk is to highlight that this property does not generalize to all compactly generated simple locally compact groups beyond Lie groups. We will discuss a concrete example, namely the group of almost automorphisms of a regular locally finite tree.

**Romain Tessera**
(ENS Lyon)

*11:30 – 12:20*

*A Banachic version of a theorem of Delorme and applications*

Delorme proved in 1977 that a unitary representation with non-trivial first cohomology of a connected solvable Lie group has a dimension 1 subrepresentation. In 2000, Shalom proved that a slightly weaker property passes to lattices. With Cornulier, we give a new geometric proof of Delorme's result, which allows us to extend it to isometric actions on any reflexive Banach space. We also enlarge the class of locally compact groups satisfying the conclusion.

**Yair Glasner** *15:00 – 15:50*
(Beer Sheva)

## *Invariant random subgroups of linear groups*

Let $G$ be a locally compact group. An invariant random subgroup of $G$ is a probability measure on the (compact) set $\mathrm{Sub}(G)$ of all closed subgroups of $G$ that is invariant under conjugation. This notion generalizes the notion of a normal subgroup as well as that of a lattice. Many properties, such as Borel's density theorem, Kesten's spectral gap theorem that are known to hold for normal subgroups and lattices can be generalized to the setting of invariant random subgroups.

In my talk I will concentrate on groups with nice geometric properties, such as linear or hyperbolic groups. I will show how the interplay between the measure preserving dynamics of the action of $G$ on $\mathrm{Sub}(G)$, and the actions of $G$ coming from its geometric nature combine to give a good understanding on the invariant random subgroups of $G$.

**Nicolas Monod** *16:30 – 17:20*

(EPFL)

*Structure of locally compact groups: discrete vs. non-discrete*

To be precise, the discrete vs. non-discrete dichotomy in the structure of locally compact groups will be debated.

# Poster session

**Chris Banks** *The $k$-closure of a group acting on a tree*
(joint work with Murray Elder and George Willis)

We describe new constructions of groups acting on trees that have Property $(P^k)$ for some positive integer $k$, which is a weaker extension of Tits' Property (P). We relate the construction to previous research and prove simplicity results similar to those proven for Property (P).

**Corina Ciobotaru** *Gelfand pairs for locally compact groups acting on Euclidean buildings*

Let $G$ be a locally compact group acting properly by type-preserving automorphisms on a Euclidean building $X$, and let $K$ be the stabiliser of a special vertex. It is known that $(G, K)$ is a Gelfand pair as soon as $G$ acts strongly transitively on $X$; this is in particular the case when $G$ is a semisimple group over a local field. We show a converse to this statement, namely: if $(G, K)$ is a Gelfand pair and $G$ acts cocompactly on $X$, then the action is strongly transitive. The proof uses the existence of strongly regular elements in $G$ and their peculiar dynamics on the spherical building at infinity.

**Annalisa Conversano** *Lie-like decompositions of definable groups*
(joint work with A. Pillay)

We study analogues of classical decompositions of Lie groups from a model-theoretic perspective.

**Jonas Deré** *Anosov diffeomorphisms on infra-nilmanifolds*

Infra-nilmanifolds play a very important role in dynamical systems, especially when studying expanding maps or Anosov diffeomorphisms. Because of the algebraic nature of these manifolds, questions about self-maps can be translated into questions about endomorphisms of their fundamental groups. In this way, it was shown by M. Gromov that every expanding map on a compact manifold is topologically conjugate to an affine infra-nilendomorphism. Up to now it is unknown if a similar statement also holds for Anosov diffeomorphisms, although some partial results point in that direction. This motivates the study of Anosov diffeomorphism on infra-nilmanifolds. On my poster I explain an easy criterion to decide whether an infra-nilmanifold modeled on a free nilpotent Lie group admits an Anosov diffeomorphism or not.

**Elisabeth Fink** *Non-trivial words in branch groups*

I will present a branch group $G$ such that $G$ has no free subgroups but exponential growth. The poster will show how, given any two elements $g, h$ from $G$ we can construct a word $w_{g,h}(x, y)$ in the free group $F(x, y)$ such that $w_{g,h}(g, h) = 1$ in $G$. I will illustrate the this construction with an explicit example.

**Ann Kiefer** *Discontinuous actions of unit groups of orders in rational group rings $\mathbb{Q}G$*

The main goal is the investigation on the unit group of an order $\mathcal{O}$ in a rational group ring $\mathbb{Q}G$ of a finite group $G$. In particular we are interested in the unit group of $\mathbb{Z}G$. Only for very few finite non-abelian groups $G$ the unit group $\mathcal{U}(\mathbb{Z}G)$ has been described, and even for fewer groups $G$ a presentation of $\mathcal{U}(\mathbb{Z}G)$ has been obtained. Nevertheless, for many finite groups $G$ a specific finite set $B$ of generators of a subgroup of finite index in $\mathcal{U}(\mathbb{Z}G)$ has been given. The only groups $G$ excluded in this result are those for which the Wedderburn decomposition of the rational group algebra $\mathbb{Q}G$ has a simple component that is either a non-commutative division algebra different from a totally definite quaternion algebra or a $2 \times 2$ matrix ring $M_2(D)$, where $D$ is either $\mathbb{Q}$, a quadratic imaginary extension of $\mathbb{Q}$ or a totally definite rational division algebra $\mathcal{H}(a, b, \mathbb{Q})$.

In some of these cases, up to commensurability, the unit group acts discontinuously on hyperbolic 2- or 3-space. Hence via the Poincaré theorem on fundamental domains, one is able to give a presentation of this group. This has been done in joint work with E. Jespers, S. O. Juriaans, A. De A. E Silva, A. C. Souza Filho. In other cases, the groups do not act discontinuously on hyperbolic space, but on a direct product of several hyperbolic 2- or 3-spaces. The aim is to generalize the theory of fundamental domains and group presentations to these cases. For the moment we have done this for the Hilbert Modular Group in joint work with A. del Río, E. Jespers.

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**Timothée Marquis** *Abstract simplicity of locally compact Kac–Moody groups*

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Complete Kac–Moody groups over finite fields are known to be compactly generated, totally disconnected, locally compact groups that are topologically simple (and non-discrete). We show that they are all abstractly simple, extending earlier results due to Carbone–Ershov–Ritter. Our strategy involves the computation of the closure of the contraction group of suitable elements, in which imaginary root subgroups play an essential role.

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**Rupert McCallum** *Locally compact groups with an essentially unique topology*

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In a recent publication [1], Linus Kramer demonstrated that a large class of semisimple Lie groups have the property that they admit just one locally compact sigma-compact Hausdorff group topology. There appears to be some evidence that other almost simple locally compact groups and their automorphism groups have this property as well. In particular, we have confirmed that this is the case for the group of rational points of an absolutely almost simple algebraic group defined over a non-archimedean local field, and also for the full automorphism group of a locally finite biregular tree. We summarise the current state of the evidence about how far this conjecture generalises, with some indication of the methods of proof.

### References


[1] Linus Kramer, The topology of a semisimple Lie group is essentially unique, *Adv. Math.* **228** (2011), 2623–2633.

**Sandip Singh** *Arithmeticity of certain symplectic hypergeometric groups*
(joint work with T. N. Venkataramana)

We will describe a sufficient condition on a pair of (primitive) polynomials that the associated hypergeometric group (monodromy group of the corresponding hypergeometric differential equation) is an arithmetic subgroup of the integral symplectic group.