Weyl distance preserving maps on spherical buildings

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(Joint work with Hendrik Van Maldeghem, Antonio Pasini, Jesse Tonnelier)

Abstract

I will give a not at all complete overview regarding the following question:

Can we reconstruct a spherical building of rank at least 3, when given only the elements of two of its types, and knowing only whether or not these are in one specific, prescribed mutual position?

Some background: One of the many ways to view a building is as a set of chambers $\mathcal{C}$ equipped with a (Weyl) distance $\delta : \mathcal{C} \times \mathcal{C} \to W$, for some Coxeter group W. An automorphism of the building is a map on $\mathcal{C}$ preserving the Weyl distance δ. One could wonder whether it suffices for a map on $\mathcal{C}$ to preserve just one Weyl distance $w_0 \in W$ to preserve all of them. We will take an incidence geometric approach and actually we will not work with chambers (opposed to [1]) but with the residues of cotype $\{i\}$ for all types i, and the induced Weyl distance. For example, in a building of type A_n with $n \geq 3$, the elements of type i are the subspaces of dimension i, $1 \leq i \leq n$, of an n-dimensional vector space; and for a map ρ to preserve a single Weyl distance, it means that there exist j, k, ℓ such that when U, V and $U \cap V$ are subspaces of respective dimensions j, k, ℓ, then the same holds for U^ρ, V^ρ and $(U \cap V)^\rho$. In general, it comes down to the above mentioned question.

Some spoilers: the A_n case was my first encounter with research, during my bachelor project [3], supervised by Hendrik. No surprises here. For types B_n, C_n and D_n, the problem is considerably harder and there are two nice classes of counter examples. For several special cases there were already results in the literature (e.g. [5, 7, 8, 9]); and in a paper with Hendrik Van Maldeghem [4] and one with Antonio Pasini [2], we dealt with the general situation, up to one special case. The exceptional cases are work in progress, recently re-initiated in the master thesis of Jesse Tonnelier, continuing the work of [6].

Keywords: spherical buildings, polar spaces, projective spaces
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