

# Compact oriented parallelisms of $\text{PG}(3, \mathbb{R})$ arising from oriented gl-stars

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## Abstract

A spread in real projective space is a set of lines such that every point lies on exactly one of them. A (topological) parallelism is a compact set of compact spreads such that every line belongs to exactly one of them. The standard example is Clifford parallelism. Betten and Riesinger constructed many other examples. Similarly, one can define parallelisms for oriented lines. Every parallelism gives rise to an oriented one. In [1] it was noticed that there are nice oriented parallelisms that do not arise in this way. In order to provide more such examples, we show here that a construction originally due to Betten and Riesinger carries over to the oriented case. This construction uses the Klein correspondence and provides regular examples, i.e. all spreads are equivalent to the complex spread. The input needed is a compact gl-star, that is, a set of lines such that every point outside the unit sphere lies on exactly one of them. The transfer of this notion and of the whole construction to the oriented case is not obvious. One key ingredient is to understand why orienting a spread as a 2-sphere amounts to the same as orienting all lines belonging to the spread in a coherent way. Another ingredient is a topological lemma which gives a criterion for bijectivity of maps lifted over a two-sheeted covering.

**Keywords:** parallelism, Klein correspondence, orientation

**MSC:** 51H10, 51A15, 51M15, 51M30

## References

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