On symplectic polar spaces

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Abstract

A polar space $\mathcal{S}$ is called symplectic if it admits a projective embedding $\varepsilon : \mathcal{S} \to \mathrm{PG}(V)$ such that the image $\varepsilon(\mathcal{S})$ of $\mathcal{S}$ is defined by an alternating bilinear form of V. The aim of this talk is to characterize symplectic polar spaces in terms of their incidence properties only. This is of special interest especially when $\mathcal{S}$ admits different (non-isomorphic) embeddings, as is the case when $\mathcal{S}$ is defined over a field $\mathbb{K}$ of characteristic 2. The main result we shall discuss is the following theorem as well as some of its consequences.

Theorem. *Let $\mathcal{S}$ be an embeddable polar space with collinearity given by $\perp$. Then, $\mathcal{S}$ is symplectic if and only if the following holds*

(*) if a hyperplane H of $\mathcal{S}$ contains $\{a,b\}^\perp$ for two non-collinear points a, b of $\mathcal{S}$, then there is $c \in \mathcal{S}$ such that $H = c^\perp$ (i.e. H is singular).

Keywords: polar spaces, embeddings, hyperplanes, hyperbolic lines
MSC: 51A50, 51B25, 51E24

References

[1] I. Cardinali, H. Cuypers, L. Giuzzi, A. Pasini, Characterizations of symplectic polar spaces, *to appear on Adv. Geom.* (arXiv:2205.14426)